AAQIL AHMED

JEE QUESTIONS : SYSTEM OF PARTICLES AND ROTATIONAL

1) Three particles A, B & C of masses m, 2m, 3m respectly placed on a smooth horizontal surface. Initially they are at rest and are connected by light inextensible strings forming an equilateral triangle of side a. Suddenly the string between A & B is cut

FIND

a) The initial acceleration of particle C
b) The initial acceleration of the COM of the system

ANSWER

$\theta = 60°$

$a_{cm} = 0$

Forces on C

→ Pull due to A : T_1
→ Pull due to B : T_2

RESULTANT

$$F_C = \sqrt{T_1^2 + T_2^2 + 2T_1T_2\cos 60°}$$

$$= \sqrt{T_1^2 + T_2^2 + T_1T_2}$$

$$T_1 = T_2 = \frac{mg}{2}$$

$$a_C = \frac{F_C}{3m} = \sqrt{\left(\frac{mg}{2}\right)^2 + \left(\frac{mg}{2}\right)^2 + \frac{mg^2}{4}}$$

$$= \frac{\sqrt{\frac{3m^2g^2}{2}}}{3m}$$

$$\Rightarrow a_C = \frac{\sqrt{3}g}{6}$$

Since A & B are connected to C

$$T_1 = ma_A, \quad T_2 = 2ma_B$$

2. A particle of mass 3m is at ~~least~~ rest on a smooth horizontal surface. It suddenly explodes into 3 fragment of m each. Two fragments move at right angles to each other with speed v & 2v.

FIND

i) speed of third fragment
ii) Total K.E of system after explosion

ANSWER

$\vec{P_i} = 0$

Frag 1 = mv along x axis

Frag. 2 = $m(2v)$ along y axis

Frag 3 : mu at unknown direction

$$\vec{P_3} = -(mv\hat{i} + 2mv\hat{j})$$

$$mu = \sqrt{(mv)^2 + (2mv)^2}$$

$$mu = mv\sqrt{5}$$

$$\Rightarrow u = \sqrt{5}v$$

K.E.

$K = \frac{1}{2}mv_1^2 + 2mv^2 + \frac{5}{2}mv^2$

$K = 5mv^2$

3. Two particles of masses m & 2m are placed on a smooth horizontal surface, connected by a light spring of nature length l & spring ~~connected~~ constant K. The system is at ~~res~~ rest. Suddenly the mass m starts losing mass at a constant rate μ in the horizontal direction towards the mass 2m with relative speed v. Find the acc. of COM.

ANSWER

Thrust

$F = \mu v$

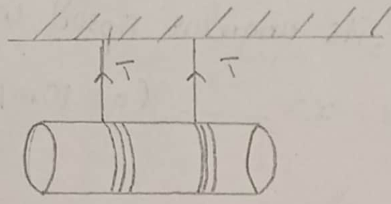
Acceleration of COM

$$M = 3m$$

$$\boxed{a_{cm} = \frac{F}{M} = \frac{\mu v}{3m}}$$

System of particles

Q A solid cylinder length is suspended symmetrically through two massless strings, as shown in the figure. The distance from the initial rest position, the cylinder should be unbinding the strings to achieve a speed of 4 m/s, is ____ cm (Take $g = 10 m/s^2$)



Sol: In case of rotational motion of a rigid body like this, the kinetic energy is not just due to the linear motion, but also due to its rotation

For a solid cylinder, $I = \frac{1}{2} m r^2$

Kinetic energy due to rotation $= \frac{1}{2} I \omega^2$

We know $\omega = \frac{v}{r}$

$\therefore$ Rotational K.E $= \frac{1}{2} \times \frac{1}{2} mv^2 = \frac{1}{4} mv^2$

Total K.E (linear + rotation) when string snaps is

$= \frac{1}{2} mv^2 + \frac{1}{4} mv^2 = \frac{3}{4} mv^2$

Equating this to P.E 'mgh' and solving for 'h' gives

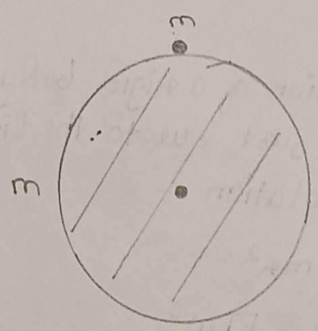
$$h = \frac{v^2}{2g} \times \frac{3}{2} = 1.2 m = 120 cm$$

Q The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the centre will be ____ m. Given, length of rod $= 10\sqrt{3} m$

Sol: $I = \frac{ML^2}{12} = MK^2 \Rightarrow K = \frac{L}{\sqrt{12}} = \frac{10\sqrt{3}}{\sqrt{12}} = 5m$

Q A disc of mass 1kg and radius R is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}$ rad/s where $x =$ ____ ($g = 10 m/s^2$)

Sol:



Loss in P.E = Gain in K.E

$$2mgR = \frac{1}{2}\left[\frac{1}{2}mR^2 + mR^2\right]\omega^2$$

$$2\cancel{m}gR = \frac{1}{2} \times \frac{3}{2}\cancel{m}R^{\cancel{2}}\omega^2$$

$$\omega^2 = \frac{8g}{3R}$$

$$\omega = \sqrt{\frac{8g}{3R}} = 4\sqrt{\frac{g}{2\times 3R}}$$

$$x = \frac{g}{2} = 5$$

System Of Particles

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Batch :- 2025-26

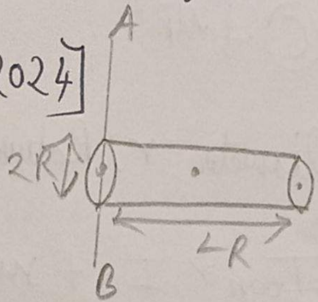
Jee Questions

Q.1) Ratio of Radius of gyration of hollow sphere to that of solid cylinder of equal mass, for moment of Inertia about their diameter axis AB as shown in the figure is $\sqrt{\frac{8}{x}}$.

The value of x is :- [Jee Main 2024]

(a) 34 (b) 51

(c) 67 (d) 17



Ans:- For hollow sphere

$$I_1 = \frac{2}{3}MR^2 = \left(\sqrt{\frac{2}{3}}R\right)^2 m$$

$$\therefore K_1 = \sqrt{\frac{2}{3}}R$$

For Solid cylinder,

$$I_2 = \frac{1}{4}mR^2 + \frac{m}{3}(4R)^2$$

$$\Rightarrow \left(\sqrt{\frac{67}{12}}R\right)^2 m$$

$$\therefore K_2 = \sqrt{\frac{67}{12}}R$$

$$\frac{K_1}{K_2} = \sqrt{\frac{8}{67}} = \sqrt{\frac{8}{x}}$$

$$\boxed{\therefore x = 67} \ (C)$$

Q.2) The moment of Inertia of a circular ring of mass m and diameter r about a tangential axis lying in the plane of the ring is :- [Jee Main 2025]

(A) $\frac{3}{8}MR^2$ (B) $2Mr^2$

(C) $\frac{1}{2}MR^2$ (D) $\frac{3}{2}MR^2$

Ans:- (*) Diameter $= r$; Radius $\Rightarrow \frac{r}{2}$

$I_{tan} \Rightarrow I_0 + mr^2$ {By Parallel axis Theorem}

$I_{tan} \Rightarrow \frac{mr^2}{2} + mr^2 \Rightarrow \frac{3}{2}mr^2$

Now; $r = \frac{r}{2}$;

$\therefore I_{tan} \Rightarrow \frac{3}{2} \times m \times \frac{r^2}{4} \Rightarrow \frac{3mr^2}{8}$

$$\boxed{\therefore I_{tan} = \frac{3mr^2}{8}}$$ (A)

Q.3) If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be :- [Jee Main 2022]

Ans:- (*) $\vec{\tau} = \vec{r} \times \vec{F}$

$\Rightarrow (2\hat{i} + \hat{j} + 2\hat{k})(3\hat{i} + 4\hat{j} - 2\hat{k})$

$\Rightarrow \underline{-10\hat{i} + 10\hat{j} + 5\hat{k}}$

System of Particles

1. A particle of mass 1 kg is moving with velocity $(2i+3j)$ m/s. It collides with another particle of mass 2kg moving with velocity $(4i-2j)$ m/s. If they stick together after collision, find the velocity of the combined particle and the loss in kinetic energy.

⇒ Applying conservation of momentum

$$m_1v_1 + m_2v_2 = (m_1+m_2)v_{cm}$$

$$(1)(2i+3j) + (2)(4i-2j) = (1+2)v_{cm}$$

$$(2i+3j) + (8i-4j) = 3v_{cm}$$

$$(10i-j) = 3v_{cm}$$

$$v_{cm} = \frac{10i-j}{3}\ m/s$$

Initial kinetic energy → $K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$= \frac{1}{2}(1)(2^2+3^2) + \frac{1}{2}(2)(4^2+(-2)^2)$$

$$= \frac{1}{2}(13) + (20)$$

$$\therefore K_i = \frac{53}{2}\ J$$

Final kinetic energy → $K_f = \frac{1}{2}(m_1+m_2)v_{cm}^2$

$$= \frac{1}{2}(3)\left(\frac{10i-j}{3}\right)^2$$

$$K_f = \frac{101}{6}\ J$$

Loss in kinetic energy → $\Delta K = K_i - K_f$

$$\Delta K = \frac{53}{2} - \frac{101}{6}\ J$$

$$\Delta K = \frac{159 - 101}{6}$$

$$\Delta k = \frac{58}{6}$$

$$\therefore \Delta K = \frac{29}{3} \text{ J}$$

$\therefore$ The final answer for velocity is $= \frac{10i - j}{3}$

$\therefore$ The final answer for loss in kinetic energy is $\frac{29}{3}$

2. A system consists of three particles A, B & C of masses 1kg, 2kg & 3kg, respectively. The velocities of A, B and C are $(2i + 3j)$ m/s, $(i - 2j)$ m/s, and $(3i + 4j)$ m/s respectively. Find the velocity of the center of mass and the total linear momentum of the system.

$\Rightarrow$ Velocity of the center of mass $\Rightarrow V_{cm} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3}$

$$V_{cm} = \frac{1(2i + 3j) + 2(i - 2j) + 3(3i + 4j)}{1 + 2 + 3}$$

$$= \frac{(2i + 3j) + (2i - 4j) + (9i + 12j)}{6}$$

$$\therefore V_{cm} = \frac{13i + 11j}{6} \text{ m/s}$$

Total linear momentum $= p = (m_1 + m_2 + m_3) V_{cm}$

$$= (1 + 2 + 3)\left[\frac{13i + 11j}{6}\right]$$

$$= 13i + 11j \text{ kg m/s}$$

$\therefore$ The final answer for velocity of center of mass is $\frac{13i + 11j}{6}$.

The total linear momentum is $13\hat{i}+11\hat{j}$.

3. A system consists of two particles of masses 2 kg and 3 kg, connected by a light rigid rod of length 5 m. The system is rotating about an axis passing through the center of mass and perpendicular to the rod. If the angular velocity is 2 rad/s, find the moment of inertia of the system.

⇒ Finding position of center of mass,
let x be the distance of the 2 kg particle from the center of mass.

$(2)x = 3(5-x)$

$2x = 15-3x$

$5x = 15$

$x = 3m$

Moment of Inertia ⇒ $I = m_1 r_1^2 + m_2 r_2^2$

$= 2(3)^2 + 3(2)^2$

$= 18 + 12$

$\therefore I = 30 \text{ kg } m^2$

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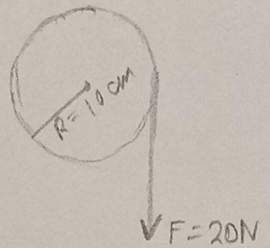
XI - A
Roll no. - 12

SYSTEM OF PARTICLES

Q1. A cord of negligible mass is wound around the rim of a wheel supported by spokes with negligible mass. The mass of wheel is 10kg and radius is 10cm and it can freely rotate without any frictions. Initially the wheel is at rest. If a steady pull of 20 N is applied on the cord, the angular velocity of the wheel, after the cord is unwound by 1m, would be:



Ans. $W = 20N(1m) = \underline{20J}$

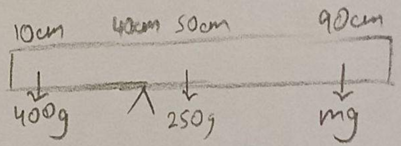
Now, $\frac{1}{2} I\omega^2 = 20J$

$I = MR^2 \Longrightarrow I = 10(0.01) = \underline{0.1 kgm^2}$

$\therefore \omega^2 = 40 \times \left(\frac{1}{10}\right) = 40 \times 10 = 400$

$\Longrightarrow \underline{\omega = 20 rad/s}$

Q.2. A uniform rod of mass 250g having length 100cm is balanced on a sharp edge at 40cm mark. A mass of 400g is suspended at 10cm mark. To maintain the balance of the rod, the mass to be suspended at 90 cm mark is:



Ans $T_{net} = 0 \Rightarrow 400(30) = (250g \times 10)(mg \times 50)$

$\therefore m = \frac{12000 - 2500}{50} = \frac{9500}{50}$

$\therefore \underline{m = 190g}$

Q3. A solid sphere is rolling without slipping on a horizontal plane. The ratio of the linear kinetic energy of the centre of mass of the plane & rotational kinetic energy is:

Ans.

$$\frac{\text{Linear KE}}{\text{Rotational KE}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2} = \frac{mv^2}{\frac{2}{5}mR^2\omega^2} \quad (v = \omega R)$$

$$= \frac{5}{2}$$

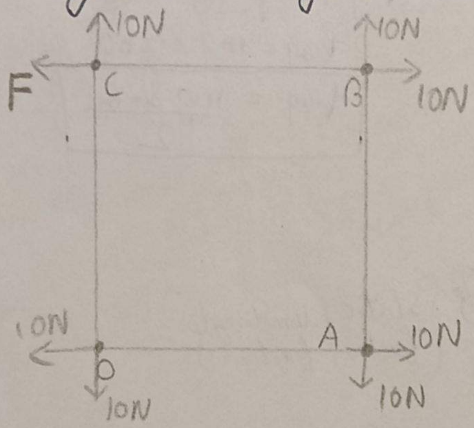
Dhruv K. Prajapati

XI A

Sarthak Shinde
2025-26

System Of Particles

Q) A Square Lamina OABC of length 10cm is pivoted at 'O'. Forces act at Lamina as shown in figure. If Lamina Stationary, then magnitude F is-



A. 20N
B. zero N
C. 10N
D. $10\sqrt{2}$ N

Sol.

Along Y-axis the net Force is zero.

Along X-axis net torque is zero

$\vec{\tau}_{net} = 0$

$0 = 10\times l + 10\times l \cancel{+10\times l} - 10\times l - F l$

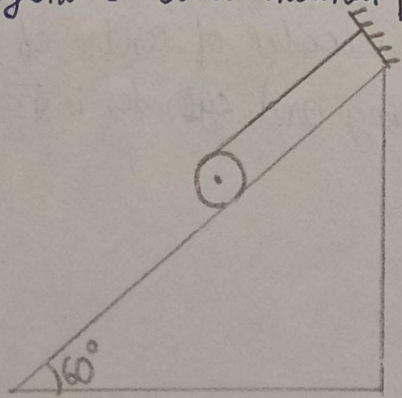
{ $l \to$ length of square/radius }

$0 = 10\times \cancel{l} - F\times \cancel{l}$

$\boxed{F = 10N}$

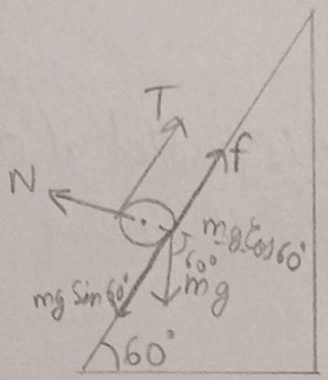
$\boxed{C.\ 10N}$

Q) A solid cylender of mass 'm' is wrapped with an inextensible light string and is placed on rough inclined plane as shown in figure. The frictional force acting between cylender and inclined plane is: (Coefficient of friction = 0.4)



(A) $\frac{7}{2}mg$
(B) $5mg$
(C) $\frac{mg}{5}$
(D) 0

Sol.



Equilibrium

$T + f = mg \sin 60°$

&

$\tau_{net} = 0$

$\Rightarrow TR - fR = 0 \Rightarrow T = f_{req}$

Since $T + f_{req} = mg \sin 60$

$f_{req} + f_{req} = mg \sin 60°$

$2 f_{req} = mg \sin 60°$

$$\boxed{f_{req} = \frac{mg \sin 60°}{2}}$$

From figure:

$N = mg \cos 60°$

$f_s = \mu N = \mu \times mg \cos 60° \; \{\mu = 0.4\}$ {Static friction / limiting friction}

$f_s = 0.4 \times mg \cos 60° = \frac{2 mg \cos 60°}{5}$

Since limiting friction $(f_s) <$ Req. friction (f_{req}),

→ So cylender not in equilibrium,

→ So friction is kinetic

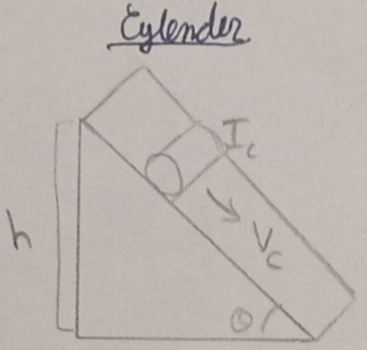
$f = \mu_k N \; \{\mu_s = \mu_k = 0.4\}$

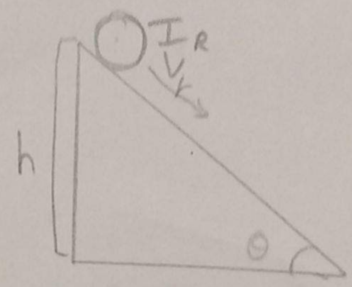
$f = 0.4 \times mg \cos 60 = \frac{4}{10} \times mg \times \frac{1}{2}$

$$\boxed{f = \frac{mg}{5}}$$ (C)

Q) Two bodies a ring & a solid cylender of same material and are rolling down without slipping an inclined plane. The radii of bodies are same. The ratio of velocities of centre of mass at bottom of inclined plane of ring and cylender is $\frac{\sqrt{x}}{2}$ Then value of x is ________.

Sol. Cylender | Ring





I about point of contact $= I_C$

$I_C = \frac{3}{2} mR^2$

I about point of contact $= I_r$

$I_r = \frac{\cancel{2}}{\cancel{2}} mR^2$

At top Potential E is equal to Kinetic E at bottom

$mgh = \frac{1}{2} I_c \omega_c^2 \left\{ \omega_c = \frac{V_c}{R} \right\}$

$mgh = \frac{1}{2} \times \frac{3}{2} m\cancel{R^2} \times \frac{V_c^2}{\cancel{R^2}}$ --- (i)

$mgh = \frac{1}{2} I_r \omega_r^2 \left\{ \omega_r = \frac{V_r}{R} \right\}$

$mgh = \frac{1}{2} \times \frac{\cancel{2}}{\cancel{2}} m\cancel{R^2} \times \frac{V_r^2}{\cancel{R^2}}$ --- (ii)

From (i) and (ii)

$\frac{1}{\cancel{2}} \frac{3}{2} \cancel{m} V_c^2 = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{2}} \cancel{m} V_r^2$

$\frac{V_r^2}{V_c^2} = \frac{3}{4}$

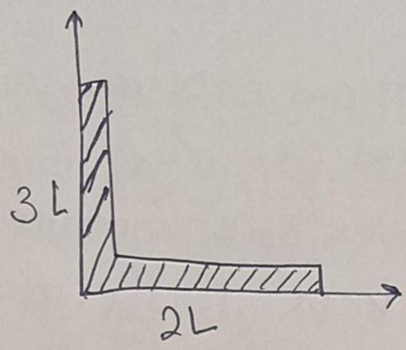
$\frac{V_r}{V_c} = \sqrt{\frac{3}{4}} = \boxed{\frac{\sqrt{3}}{2}} = \frac{\sqrt{x}}{2}$

$\boxed{x = 3}$

Abhishek

System of Particles

1. Rod of lenght 5L is bent right angle keeping one side lenght as 2L.



The position of the centre of mass of the system:
[consider L=10 cm]

Answer:-

Horizontal Rod:

Lenght $= 2L$

Mass $\propto 2L$.

centre at:

$(x_1, y_1) = (L, 0)$

Vertical Rod:-

lenght $= 3L$

Mass $\propto 3L$

centre at:

$(x_2, y_2) = (0, \frac{3L}{2})$

$\therefore$ CoM:-

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{2L(L) + 3L(0)}{5L} = \frac{2L}{5} = 4\,cm$$

$$y_{cm} = \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2} = \frac{0(2L) + (3L/2)(3L)}{5L} = \frac{9L}{10} = 9\,cm$$

$\therefore$ Answer:-

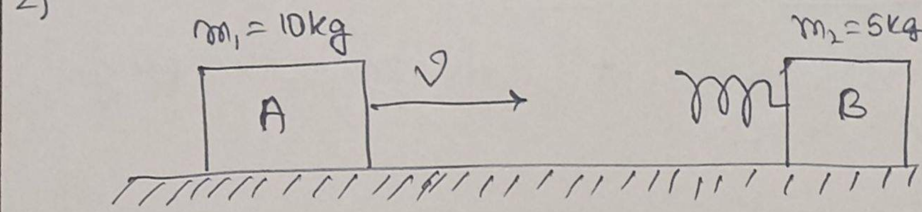
$x_{cm} = 4\,cm$

$y_{cm} = 9\,cm$.

$(x, y) = (4, 9)$

$\Rightarrow 4\hat{i} + 9\hat{j}$.

2)



Consider two blocks A and B of masses $m_1 = 10kg$ and $m_2 = 5kg$ that are placed on a frictionless table. The block A moves with constant speed $v = 3\,m/s$ towards the block B kept at rest. A spring with spring constant $K = 3000 N/m$ is attached with the block B as shown in the fig. After the collision, suppose that the block A and B, along with the spring in constant compression state, move together, then the compression in the spring is, (Neglect the mass of the spring).

Answer:-

For A,

$p_1 = m_1 v = 10 \times 3 = 30$

$\therefore P_{common} = m_{total} \times v_{final}$

$$\Rightarrow v_{final} = \frac{P_{common}}{m_{total}} = \frac{30}{15} = 2\,m/s.$$

~~$\therefore$ Energy stored in spring = $KE_i - KE_f$.~~

$$\cancel{= \frac{1}{2}\left[m_1 v^2 - m_{total} v_{final}^2\right] = \frac{1}{2}\left[90 - 60\right]}$$

$$\cancel{= 15 J.}$$

$\therefore$ Energy stored in Springs $= K.E_i - K.E_f$

$$= \frac{1}{2}\left[m_1 v^2 - m_{total}(V_{final})^2\right]$$

$$= \frac{1}{2}[90-60] = \frac{1}{\cancel{2}} \times \overset{15}{\cancel{30}} = 15\,J.$$

$\therefore \frac{1}{2}kx^2 = 15 \Rightarrow \frac{1}{2} \times 3000 \times x^2 = 15.$

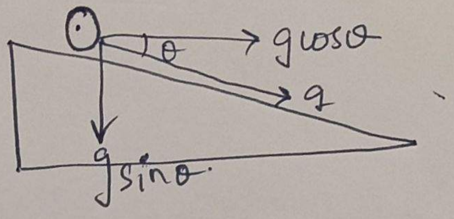
$\Rightarrow x^2 = \frac{15 \times 2}{3000} \Rightarrow x = \sqrt{\frac{\cancel{30}}{\underset{100}{\cancel{3000}}}} \Rightarrow x = \frac{1}{10}$

$= 0.1m$

$= 10cm.$

$\boxed{\text{(B) } 0.1m}$

(3) A circular disc reaches from top to bottom of an inclined plane of lenght (l). When it slips down the plane, it takes t secs. When it rolls down the plane then it takes $\left(\frac{\alpha}{2}\right)^{1/2}$ ts, where α is ______.



$\therefore$ Case 1:-

Time $= t.$

Case 2:-

Time $= \sqrt{\frac{a}{2}}\,t.$

$a_s = g\sin\theta.$

$\therefore l = \frac{1}{2}at^2.$

$= \frac{1}{2} \times g\sin\theta \times t^2 \Rightarrow t^2 = \frac{2l}{g\sin\theta}.$

Case 2:-

$t = \sqrt{\frac{\alpha}{2}}\, t.$

$\therefore I = \frac{1}{2} MR^2.$

$\therefore a_r = \dfrac{g \sin\theta}{1 + \frac{I}{MR^2}} = \dfrac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3} g \sin\theta.$

$\therefore l = \frac{1}{2} a t^2 = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} g \sin\theta \times t_r^2 = \frac{1}{3} g \sin\theta\, t_r^2.$

$\therefore t_r^2 = \dfrac{3l}{g\sin\theta}.$

$\therefore \dfrac{t_r^2}{t^2} = \dfrac{3}{2}. \Rightarrow t_r = \sqrt{\frac{3}{2} t^2} = t\sqrt{\frac{3}{2}}$

$\Rightarrow \sqrt{\frac{\alpha}{2}} = \sqrt{\frac{3}{2}} \Rightarrow \boxed{\alpha = 3}$

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

(1) A uniform rod of length L and mass M is pivoted at one end and held vertically. It is allowed to fall freely under gravity. The angular velocity of the rod when it reaches the horizontal position is:-

(A) $\sqrt{\frac{3g}{L}}$ (b) $\sqrt{\frac{2g}{L}}$ (c) $\sqrt{\frac{g}{L}}$ (d) $\sqrt{\frac{6g}{L}}$

Ans:- initial PE of rod's centre of mass $= Mg \cdot \frac{L}{2}$

PE at horizontal $= 0$

Loss in PE $= \frac{MgL}{2}$

This becomes rotational KE

$$\frac{1}{2} I\omega^2 = \frac{MgL}{2}$$

for a rod pivoted at one end $I = \frac{1}{3}ML^2$

Substituting

$$\frac{1}{2} \cdot \frac{1}{3} ML^2 \cdot \omega^2 = \frac{MgL}{2}$$

$\omega^2 = \frac{3g}{L}$ $\omega = \sqrt{\frac{3g}{L}}$ Answer :- option (A)

(2) Two particles of masses $m_1 = 2kg$ and $m_2 = 1kg$ are located at positions $x_1 = 0m$ and $x_2 = 6m$ on the x-axis. The position of the center of mass of the system is

(A) 2m (B) 3m (C) 4m (D) 5m

Answer:-

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{6}{3} = 2m$$

option (A)

(3) A disc of mass M and radius R is rolling without slipping with velocity v. The kinetic energy of the disc is

(A) $\frac{1}{2}Mv^2$ (B) Mv^2 (C) $\frac{3}{4}Mv^2$ (d) $\frac{1}{4}Mv^2$

Answer:- Total KE = Translational KE + Rotational KE

Translational KE = $\frac{1}{2}Mv^2$

Rotational KE = $\frac{1}{2}I\omega^2$

for disc $I = \frac{1}{2}MR^2$

Rolling condition $\omega = \frac{v}{R}$

Rotational KE = $\frac{1}{2}\cdot\frac{1}{2}MR^2\cdot\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$

Total KE = $\frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2$

Option (C) $\frac{3}{4}Mv^2$

Arun Sam Isac
2025-26

System of Particle

Q.1 A child is standing with folded hands at the centre of a platform rotating about its central axis. The K.E of the system is K. The child now streches his arms so that the moment of Inertia of the system doubles. The K.E of the system now is.

Ans: According to the conservation of angular momentum

$I\omega_0 = I_1\omega_1$

So we have $I_1 = 2I$

$\omega_1 = \frac{\omega_0}{2}$

Intial K.E $= \frac{I\omega_0^2}{2} = K$

Final K.E $= \frac{I_1\omega_1^2}{2} = 2I\left(\frac{\omega_0}{2}\right)^2 = \frac{K}{2}$

Q.2 A ball of mass of 0.5 kg is attached to the end of a string having a length 0.5m. The ball is rotated on a horizontal circular path about a vertical axis.

Q.2

The maximum tension that the string can bear is 324N. The maximum possible value of the angular velocity of the ball is.

Ans: $m\omega^2 L \sin\theta \cos\theta = mg \sin\theta$

$\cos\theta = \frac{g}{\omega^2 L}$, $\sin\theta = \frac{1}{\omega^2 L}$

$\sin\theta = \frac{1}{\omega^2 L} \sqrt{(\omega^2 L)^2 - g^2}$

$T = mg \cos\theta + m\omega^2 L \sin^2\theta$

$T = mg\left(\frac{g}{\omega^2 L}\right) + \left(\frac{m\omega^2}{(\omega^2 L)^2}\right)((\omega^2 L)^2 - g^2)$

$T = \frac{mg}{\omega^2 L}(g^2 + (\omega^2 L)^2 - g^2)$

$T = m\omega^2 L = 324$

$\omega = \sqrt{\frac{324}{0.5 \times 0.5}}$

$\omega = 36$ rad/s

Q.3 The moment of intertia of a uniform cylinder of length and Radius R about its perpendicular bisector is I. What is the ratio I/R such that the moment of Intevia is minimum?

$$I = m(I^2/12 + R^2/4)$$

Volume = $\pi R^2 I$

$$I = \frac{m}{4}(V/\pi I + I^2/3)$$

$$\frac{dI}{dn} = \frac{m}{4}(-V/\pi I^2 + 2I/3) = 0$$

$$\frac{m}{4}(-V/\pi I^2 + 2I/3) = 0$$

$$V/\pi I^2 = 2I/3$$

$$R^2/I = 2I/3$$

$$I^2/R^2 = 3/2$$

$$\frac{I}{R} = \sqrt{\frac{3}{2}}$$

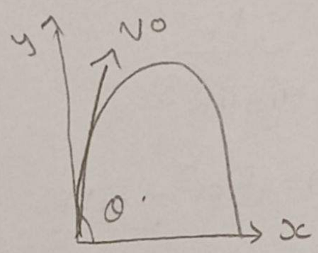
2025-2026
Zaid Shamsheeri
XI-A.

System of particles.

Q1. Small projectile of mass "m" is projected at an angle θ with a x-axis with an initial velocity V_0 in the x-y plane as show

$t < \frac{v_0 \sin\theta}{g}$, find angular momentum

Sol:



$$t < \frac{v_0 \sin\theta}{g}$$

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$x = v_0 \cos\theta t$$

$$y = v_0 \sin\theta t - \tfrac{1}{2} g t^2$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$v_x = v_0 \cos\theta$$

$$v_y = v_0 \sin\theta - gt$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$y v_x = (v_0 \sin\theta t - \tfrac{1}{2} g t^2)(v_0 \cos\theta)$$

$$x v_y - y v_x = \tfrac{1}{2} g v_0 t^2 \cos\theta$$

$$\boxed{\vec{L} = -\tfrac{1}{2} m g v_0 t^2 \cos\theta \,\hat{k}}$$

Q2. Distance of centre of mass of a solid uniform cone from its vertex is z_0. If radius of its base "R" and height is h then z_0 is?

Sol:

$$dV = \pi r^2 dz = \pi \left(\frac{R}{h}\right)^2 z^2 dz$$

$$dm = \rho dV = \rho\pi \left(\frac{R}{h}\right)^2 z^2 dz$$

$$\int dm = \rho\pi \left(\frac{R}{h}\right)^2 \int_0^h z^2 dz \Rightarrow \rho\pi \left(\frac{R}{h}\right)^2 \cdot \frac{h^3}{3}$$

$$\int z\,dm = \rho\pi \left(\frac{R}{h}\right)^2 \cdot \frac{h^4}{4}$$

$$\boxed{z_0 = \frac{\frac{h^4}{4}}{\frac{h^3}{3}} = \frac{3h}{4}}$$

Q. from a solid sphere mass "M" and radius "R" a cube of maximum possible volume is cut. moment of Inertia of cube about an axis passing through its centre and ~~perd~~ perpendicular to one of its faces is .

Sol:

$$a\sqrt{3} = 2R \Rightarrow a = \frac{2R}{\sqrt{3}}$$

$$\rho = \frac{M}{4/3\pi R^3}$$

$$m = \rho a^3 = \frac{2M}{\sqrt{3}\pi}$$

$$I = \frac{1}{6} Ma^2$$

$$I = \frac{1}{6} \cdot \frac{2M}{\sqrt{3}\pi} \cdot \frac{4R^2}{3} = \boxed{\frac{4MR^2}{9\sqrt{3}\pi}}$$

D.S. HARINI, XI-A

2025-2026

System of Particles

Q. A uniform circular disc of radius 'R' and mass 'M' is rotating about an axis perpendicular to its plane & passing through centre. A smaller circular part of root. R/2 is removed from original disc as shown ~~to~~ in figure. Find moment of inertia of remaining part of original disc about axis given above.

Sol: $m = \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2$

$m = \frac{M}{4}$

$I = I_1 - I_2$

where I_2 = MOI of whole disc about perpendicular axis through O

I_2 = MOI of removed disc about ⊥ axis

$I = \frac{MR^2}{2} - \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$

$I = \frac{MR^2}{2} - \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right)$

$I = MR^2\left[\frac{1}{2} - \frac{1}{32} - \frac{1}{16}\right] = \frac{13}{32} MR^2$

$I = \frac{13}{32} MR^2$

Q. A carpet of mass M made of inextensible material is rolled along its length in form of a cylinder of radius R and its kept on rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2.

Sol: The radius is reduced to R/2 the mass of the rolled carpet $= \frac{M}{\pi R^2 l} \times \left(\frac{R}{2}\right)^2 l = \frac{M}{4}$

Release of PE $= MgR - \left(\frac{M}{4}\right)\frac{gR}{2} = \frac{7}{8} MgR$

The K.E at this instant is:

$$\frac{1}{2}(Mv^2) + \frac{1}{2} I\omega^2 \Rightarrow \frac{1}{2}\frac{M}{4}v^2 + \frac{1}{2}\frac{MR^2}{32}$$

$$\left(\frac{2v}{R}\right)^2 \Rightarrow \left(\frac{3}{16}\right) Mv^2$$

$$\left(\frac{3}{16}\right) Mv^2 = \frac{7}{8} MgR$$

$$v = \sqrt{\frac{14}{3} gR}$$

Q. A rod of linear mass density 'λ' & length (l) is bent from a ring of radius 'R' moment of inertia of ring about any of its dimension.

$l = 2\pi R$

$M = \lambda \times L$

$I = \frac{MR^2}{2}$

$I = \frac{\lambda \times L}{2} \times \left(\frac{l}{2\pi}\right)^2$

$I = \frac{\lambda L^3}{8\pi^2}$ ← ans

Q) A particle rotating in circular path and at any instant its motion can be described as $\theta = \frac{5t^4}{40} - \frac{t^3}{3}$. The angular acceleration (α) at $t = 10$ is ____ rad/s^2

(A) 150
(B) 130
(C) 120
(D) 170

Ans: $\theta = \frac{5}{40} t^4 - \frac{t^3}{3}$

$$\omega = \frac{d\theta}{dt} = \frac{20t^3}{40} - \frac{3t^2}{3}$$

$$\alpha = \frac{d\omega}{dt} = \frac{2\cancel{0} \times 3t^2}{2\cancel{4}\cancel{0}} - \frac{\cancel{3} \times 2t}{\cancel{3}}$$

$$\alpha = \frac{3}{2}t^2 - 2t$$

$$\alpha\Big|_{t=10} = \frac{3 \times \cancel{10}^{5} \times 10}{\cancel{2}} - 20 = 150 - 20 = \boxed{130}$$

Q) When position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ changes sign as $-\vec{r}$, which following vector will not flip under sign change.

(A) Angular momentum
(B) Velocity
(C) Linear momentum
(D) Acceleration.

Ans → (A) Angular momentum because its axial vector.

Q.) A Solid sphere of $r = 10cm$ is rotating about an axis which is at a distance 15cm from its centre. The radius of gyration about this axis $\sqrt{n}$ cm

$n =$ ________ .

Ans :

$$K^2 = K_{cm}^2 + d^2$$

$$I_{centre} = \frac{2}{5} MR^2 \text{ so } K_{cm}^2 = \frac{2}{5} R^2$$

with $R = 10cm$ and $d = 15cm$

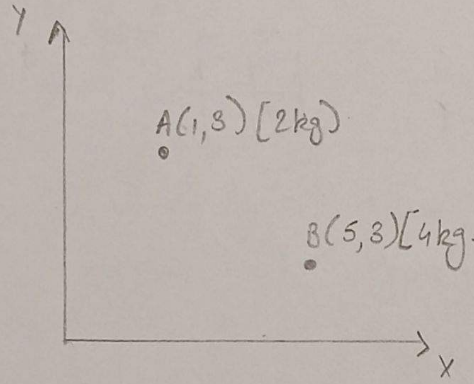
$$K^2 = \frac{2}{5}(10^2) + (15)^2 = 265$$

$$K = \sqrt{265} \qquad \boxed{n = 265}$$

System of Particles.   SAANVI

Q Two particles of masses 2kg and 4kg are placed at point A(1, 3) and B(5,3) respectively. Find the coordinates of centre of mass of the sys.

(JEE Main 2019 pattern)



$$\Rightarrow x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

x - coordi. :

$$x_{cm} = \frac{2(1) + 4(5)}{2+4}$$

$$= \frac{22}{6} = \boxed{\frac{11}{3}}$$

y-coordi. :

$$y_{cm} = \frac{2(3) + 4(3)}{6}$$

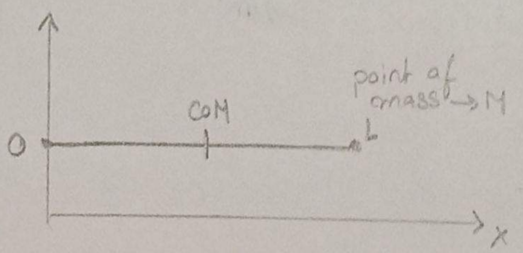
$$= \frac{18}{6} = \boxed{3}$$

Ans $\Longrightarrow \left(\frac{11}{3}, 3\right)$.   CoM shifts to heavier mass.

Q A uniform rod of length L and mass M lies along x-axis from $x = 0$ to $x = L$. A point mass M is attached at $x = L$

Find the position of the centre of Mass

(JEE Main 2020 PYQ type)



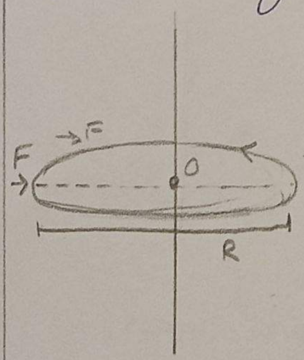
Mid point of rod $= L/2$.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{cm} = \frac{M(L/2) + M(L)}{2M}$$

$$\rightarrow \frac{ML/2 + ML}{2\cancel{M}} = \frac{\frac{3ML}{2}}{2M} = \frac{3\cancel{M}L}{4\cancel{M}} = \boxed{\frac{3L}{4}} //.$$

Q. A uniform solid disc of Mass M and Radius R is free to rotate about a fixed vertical axis passing through its centre.
A constant force F is applied tangentially at the rim of the disc.
Find the angular acceleration produced in the disc.



Force and radius is perpendicular

$\tau = rF\sin\theta$ $\qquad r = R$

$\tau = FR$ $\qquad \theta = 90^\circ$

Solid disc $\longrightarrow I = \frac{1}{2}MR^2$

$\tau = I\alpha$

$FR = \frac{1}{2}MR^2 \cdot \alpha \quad = \quad \alpha = \frac{F\cancel{R}(2)}{MR^{\cancel{2}}}$

$$\boxed{\alpha = \frac{2F}{MR}} //.$$

Deepinder

Q1) A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30N? What is the linear acceleration of the roape?

A) Given, $m = 3kg$, $r = 40cm = 0.4m$, $F = 30N$

Hollow cylinder: $I = mr^2 = 3 \times 0.4^2 = 0.48\ kgm^2$ //

Rope: $\tau = Fr = 30 \times 0.4 = 12 Nm$

$\tau = I\alpha$

$\Rightarrow \alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25\ rads^{-2}$

Rope: $a = r\alpha = 0.4 \times 25 = 10 ms^{-2}$ //

Q2) A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to $10\pi\ rads^{-1}$! Which of two will start to roll first? $(\mu_k = 0.2)$.

A) Given: $r = 10\ cm = 0.1m$, $V_0 = 0$, $\mu_k = 0.2$, $\omega_0 = 10\pi\ rads^{-1}$

$F = ma \Rightarrow \mu_k mg = ma$, where $a = \mu_k g$

$V_1 = U + at$

$V_1 = 0 + \mu_k gt$

$V_1 = \mu_k gt$

τ about centre of mass $= fr = I\alpha$

$\alpha = \frac{\mu_k mgr}{I}$

$\omega_1 = \omega_0 - \alpha t = \omega_0 - \frac{\mu_k mgr}{I} t$, (-ve) since opposing motion

$V_1 = \omega_1 r$

$\mu_k gt = \omega_0 r - \frac{\mu_k mgr^2}{I} t$

$t = \frac{\omega_0 r}{\mu_k g(1 + \frac{mr^2}{I})}$

Disk: $I = \frac{1}{2}mr^2$

$t_D = \frac{\omega_0 r}{\mu_k g(1 + \frac{mr^2}{\frac{1}{2}mr^2})} = \frac{\omega_0 r}{3\mu_k g}$

Ring: $I = mr^2$

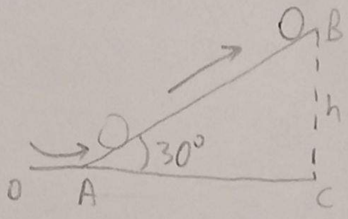
$t_r = \frac{\omega_0 r}{\mu_k g(1 + \frac{mr^2}{mr^2})} = \frac{\omega_0 r}{2\mu_k g}$

$t_D < t_r$ // $\therefore$ Solid disc rolls first

Q3) A Solid cylinder rolls up an incline plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of $5 ms^{-1}$. How far will the cylinder go up to the plane? How long will it take to return to the bottom?

A) Given, $v = 5ms^{-1}$, $\theta = 30°$

At A, Energy of cylinder:



$K.E_{rot} = K.E_{trans}$

$\frac{1}{2} I\omega^2 = \frac{1}{2} mv^2$

At B, Energy of cylinder $= mgh$

Law of conservation of energy:

$\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = mgh$

Cylinder: $I = \frac{1}{2} mr^2$

$\frac{1}{4} mr^2\omega^2 + \frac{1}{2} mv^2 = mgh$

$\frac{1}{4} r^2\omega^2 + \frac{1}{2} v^2 = gh$

$\frac{1}{4} v^2 + \frac{1}{2} v^2 = gh \quad (v = r\omega)$

$\frac{3}{4} v^2 = gh$

$h = \frac{3}{4} \frac{v^2}{g} = \frac{3}{4} \times \frac{25}{9.8} = 1.91\ m$

In ΔABC,

$\sin\theta = \frac{BC}{AB}$

$\sin 30 = \frac{h}{AB}$

$AB = \frac{1.91}{0.5} = 3.82\ m$

$\therefore$ Cylinder will move 3.82m up the inclined plane.

While rolling back,

$v = \left(\frac{2gh}{1+\frac{K^2}{R^2}}\right)^{\frac{1}{2}} = \left(\frac{2g\,AB\sin\theta}{1+\frac{K^2}{r^2}}\right)^{\frac{1}{2}} = \left(\frac{2g\,AB\sin\theta}{1+\frac{1}{2}}\right)^{\frac{1}{2}} = \left(\frac{4}{3} g\,AB\sin\theta\right)^{\frac{1}{2}}$

$[h = AB\sin\theta]$ $\left[K^2 = \frac{R^2}{2}\right]$

$t = \frac{AB}{v}$

$= \frac{AB}{\left(\frac{4}{3} g\,AB\sin\theta\right)^{\frac{1}{2}}}$

$= \left(\frac{3AB}{4g\sin\theta}\right)^{\frac{1}{2}}$

$= 0.764\ s$

$\therefore$ Time taken to return to bottom $= 0.764s * 2 = 1.53\ s$